Feedback and Motivation in Dynamic Tournaments

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We investigate the choice to conduct interim performance evaluations in a dynamic tournament. When a worker's ability does not influence the marginal benefit of effort, the choice depends on the shape of the cost of effort function. When effort and ability are complementary, feedback has several competing effects: it informs workers about their relative position in the tournament (evaluation effect) as well as their relative productivity (motivation effect) and it creates signal-jamming incentives to exert effort prior to the performance evaluation. These effects suggest a tradeoff of performance feedback between evaluation and motivation which is in accordance with organizational behavior research and performance appraisal practices.

1. Introduction

This paper studies interim performance appraisals. Interim performance evaluations occur at some point, usually midway, through the completion of a task when evaluators inform individuals of their progress toward achieving a specific goal, for example, a promotion. In particular, we are interested in how organizations decide whether or not to provide feedback to their workers on how their performance to date has been evaluated and how the choice of feedback policy affects the effort choice and performance of workers.

Feedback policies are pervasive in organizational settings. Citing a series of human resource studies, Murphy and Cleveland (1995)

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document that between 74% and 89% of business organizations have a formal performance appraisal and feedback system. DeVries et al. (1986) note that since the 1960s performance appraisals were increasingly used for employee development and feedback. In fact, in almost all organizations, at least some information is revealed to workers at regular intervals about how well they have performed in the past. Companies in which promotion decisions constitute a large part of rewards, such as law firms or consulting firms, inform their workers about their previous performance and prospects long before the actual promotion decisions are made. In universities, junior faculty members are given feedback through formal review processes about past performance and their prospects of obtaining tenure. Midterm exams that count toward the final grade that students receive for a particular course are another example of a case where information about performance is revealed to students in advance of the final results.

There are also many other examples of informational feedback mechanisms outside of organizations. In patent races, the governing body has to decide whether or not to force companies to reveal how much progress they have made toward a particular discovery. In sports competitions, much attention is devoted to the design of the information feedback scheme. For example, the games in the final leg of any soccer league competition are usually held at the same time so that vital information is obscured to the contestants whereas contestants in a race are provided with information about split times relative to other competitors or previous records. Finally, feedback mechanisms such as opinion polls may also influence candidates' and voters' decisions in political elections. Until recently, Austrian law prohibited the publication of opinion polls in the last week before an election whereas other countries allowed information to be released to the public.

Despite their pervasiveness, interim performance appraisals have, until recently, received scant attention in the economic literature. Given the widespread use of interim performance appraisals, it is even more surprising that the previous contributions that deal with the issue of feedback in dynamic moral hazard problems cast a pessimistic light on the role of interim evaluations. In fact, they mostly provide convincing explanations for why we should not observe interim evaluations in practice (Lizzeri et al., 2002; Fuchs, 2007).[1] There is, however, a large organizational behavior and human resource management literature that discusses the effects of organizational feedback mechanisms. In a number of studies (Stone and Stone, 1985; Ashford, 1986; Locke

1. However, Manso (2007) shows that feedback is crucial when the principal wants to incentivize workers to innovate.

and Latham, 1990) feedback is found "to provide critical input for forming realistic self-assessments in the work setting" and "is a key to maintaining high levels of work motivation" (Murphy and Cleveland, 1995, p. 328). Furthermore, there is evidence that the introduction of feedback mechanisms can both enhance and lower performance (Liden and Mitchell, 1985; Podsakoff and Farh, 1989) and that interim performance evaluations generally involve a tradeoff between the conflicting objectives of "development" and "evaluation" (Beer, 1987).

The negative verdict on interim performance evaluations in the economic literature and lack of consensus in the human resource literature about the desirability of interim performance appraisals may be linked to the multiplicity of effects that such mechanisms can have. First, performance appraisals affect the incentives of workers to exert effort after information has been revealed to them. This is true irrespective of whether the information that the firm transmits in such performance appraisals is given exogenously or is generated endogenously by the workers' previous output levels. In either case, the subsequent contest between workers will be biased in favor of the worker that received positive feedback (*evaluation effect*). Second, interim performance evaluations affect the morale of workers as well as the confidence they have in their skills as they learn about their productivity (*motivation effect*). Third, feedback mechanisms can affect workers' incentives to exert effort before information is revealed to them. Additional *implicit incentives* can arise because workers choose their actions strategically so as to influence the content of the interim performance appraisal. Finally, providing feedback on performance helps workers do their jobs or plan their futures better by giving them better information on which to base their decisions. In this way, interim performance appraisals affect how well workers tailor their effort choices to their ability level (*sorting effect*). Although the previous economic literature has focused on the first of these forces which we call the evaluation effect, the present paper is the first to formalize the other effects in a dynamic tournament setting and to analyze the optimal design of a feedback policy in their presence.

Tournaments are ubiquitous in economic organizations because performance is often rewarded by promoting high-performing employees. Therefore, the most natural setting in which to explore the question of interim performance evaluations in organizations is that of a contest between workers who compete for a fixed prize, for example, a job promotion. First, such tournaments are inherently dynamic as during the contest the principal often observes some interim performance measures that she can use to provide feedback to the agents. Second, workers' performance is generally private information of the manager or the firm's personnel division. Workers, on the other hand, cannot fully observe their own performance because output can often only be measured by a combination of factors such as social skills, originality, team-working ability and because there may be a significant subjective element in the evaluation of performance. Furthermore, organizations usually have much more experience in assessing the contributions of an individual worker who tends to have little experience with the tasks he is performing. Third, workers generally also differ in terms of their ability as well as the confidence they have in their skills.

We therefore study a two-period tournament model in which two risk-neutral agents make private effort choices but cannot observe their performances due to exogenous noise and incomplete knowledge of their own ability levels. The principal (she) is assumed to be risk-neutral and solely interested in maximizing expected total performance of the agents over the two periods. Furthermore, the principal privately observes the difference in agents' performance realizations after the first stage and then chooses between the following two alternatives. In a *no-feedback* scenario, she does not transmit any of her private information to the agents so that when choosing second-period effort the agent does not know the first-period outcome. In a *full-feedback* scenario, the principal truthfully and publicly reveals the first-period output difference to the two contestants.[2] Under such a policy the agents learn their relative standing before choosing second-period efforts. The optimal feedback policy is one that maximizes the expected output of the two agents over the two periods.

We begin the analysis by exploring the case in which the performance of each agent in each period is the sum of his privately chosen effort, a persistent ability component and a random noise term. This setup allows for heterogeneity in ability among contestants as well as learning about ability levels on behalf of the contestants when they receive an interim performance evaluation. The optimal feedback policy takes a simple form which crucially depends on the functional form of the disutility of effort. If the marginal cost of effort is concave, a full-feedback policy elicits higher expected second-period effort and output from the agents than a no-feedback policy. If the marginal cost of effort is convex, the opposite relation holds. Under the common assumption of linear marginal cost of effort both feedback policies yield the same expected effort and output levels.

To examine the effects of organizational feedback mechanisms on motivation and morale, we enrich the basic model by assuming

2. Note, however, that there might be circumstances under which the principal may find it more appropriate to send private messages to the agents.

that effort and ability are complementary so that beliefs about ability directly impact the effort choice of agents. We identify several effects of interim performance evaluations that are emphasized in the human resource management and organizational behavior literature, yet were previously not documented in the economic literature. We show that the two different feedback policies will lead to different expected effort and performance levels even when the marginal disutility of effort is linear. As before when information about relative past performance is revealed, workers learn about ability. However, because effort and ability are complementary, their beliefs about ability directly influence the workers' perceived marginal benefit of effort. Workers will therefore tailor their second-period effort choices to the beliefs they hold about their ability level. We call this result the *motivation effect* of interim performance evaluations because it motivates contestants who (correctly) believe themselves to be more able to exert more effort. However, this motivation effect is a double-edged sword because it also discourages effort exertion by less able contestants. Yet, the efficient sorting that results from this motivation effect leads to higher expected second-period output than if no information is revealed to the contestants because more able contestants exert more effort whereas less able contestants exert less effort. Furthermore, we demonstrate that a procedure that gives full feedback to agents creates *implicit incentives* prior to the revelation of information that are absent in the no feedback case. These implicit incentives arise because, in a model where effort and ability are complementary, each contestant would like to make his opponent believe that his own ability (i.e., marginal benefit of effort) is higher than it actually is. This can be achieved by increasing first-period effort and therefore biasing the first-period output difference in one's favor. As a result, first-period effort and output are higher under a full-feedback policy. However, all these beneficial effects will have to be weighed against the adverse consequences that result from the asymmetries that a full-feedback policy generates whenever it reveals that the first-period output difference is in favor of one of the contestants. Equilibrium effort in the second-period contest is highest when the contestants are close to each other in terms of first-period output, but effort is decreasing as the difference in first-period outputs grows large and thus strongly favors one of the agents. This is the *evaluation* or *lack-of-competition effect* which has been the focus of the previous literature and it will tend to reduce the benefits of interim performance evaluations. Our analysis suggests a fundamental tradeoff between evaluation and motivation effects that firms face when deciding whether and how to provide interim performance evaluation.

Although tournaments have received significant attention in personnel economics (Lazear and Rosen, 1981; Green and Stokey, 1983), earlier contributions have mostly focused on the static case of one-shot interaction where contestants make a single effort choice. However, tournaments are often dynamic in nature (Rosen, 1986; Meyer, 1991, 1992). More recently, several contributions have studied the effect of information release on incentives in dynamic tournament settings (Lizzeri et al., 1999, 2002; Yildirim, 2005; Aoyagi, 2007; Gershkov and Perry, 2009; Goltsman and Mukherjee, 2008). These papers show that the release of interim information creates endogenous asymmetries between contestants and that the information revelation policy can affect effort choices both before and after the release of information.

Gershkov and Perry (2009) study midterm reviews in a two-period setting without ability heterogeneity. In contrast to the present paper that explores the question of optimal information release, they analyze whether it is optimal to give equal importance to the performance of contestants in the two periods. Goltsman and Mukherjee (2008) also investigate a dynamic tournament setting without heterogeneity in ability and binary output, but allow for more general feedback policies. Interestingly, they show that the principal is strictly better off using a partial disclosure policy that does not reveal all information. Finally, Lizzeri et al. (1999) and Aoyagi (2007) are most closely related to the present paper. Both contributions explore dynamic tournament models in which agents do not vary with respect to ability and, like in our paper, contrast incentive effects that are present under the polar opposites of no- and full-feedback. They derive conditions under which the full-feedback or no-feedback policies are optimal and that the choice in favor of interim performance appraisals depends on the third derivative of the cost of effort function.[3]

3. There are interesting similarities of the effect of information release in the literature on all-pay auctions (Baye et al., 1993; Morgan and Krishna, 1997; Siegel, 2009) which also studies settings where bidders make irreversible investments (bids) to obtain a prize (good). In such auctions, bidders may be asked to submit sealed or open bids. Because bidders learn less about their relative standing during the auction with sealed bids than under open bidding, these two settings may be compared to the no- and full-feedback scenarios studied in the dynamic tournaments literature. If the assumptions of revenue equivalence are satisfied (risk neutrality, independent private signals, efficient allocation of the good) bidders will make the same expected payments, a result that is similar to the effort equivalence result under linear marginal effort costs in the dynamic tournaments literature. However, the linkage principle (Milgrom and Weber, 1982) asserts that when bidders' signals are affiliated, the seller's expected revenue is higher in open-bid than sealed-bid second-price auctions. This suggests that interim information transmission may be beneficial also in other auction settings. Unfortunately, such a result does not exist for open- and sealed-bid all-pay auctions.

However, we show that this conclusion relies on symmetry properties that are also present in our paper if there are no ability differences or if ability enters additively into the agents' production function, but the effort equivalence result fails to hold once learning about ability also influences subsequent effort choices. Our paper further demonstrates that if contestants can learn about their abilities and ability influences the marginal benefit of effort, interim information release has important motivation and sorting effects which need to be taken into account when deciding whether and how to provide interim performance appraisals.[4]

The remainder of the paper is structured as follows. In Section 2, we present the general model. Section 3 analyzes the role of interim performance evaluation when agents do not differ with respect to ability and when ability is additive so that the motivation effect is absent. Section 4 demonstrates that the conclusions about the usefulness of organizational feedback mechanisms change dramatically once one allows for complementarity between effort and ability. Finally, in Section 5 we suggest how the present model could be extended and conclude.

2. The Model

Consider a tournament for a fixed prize between two risk-neutral agents $i = A, B$ which takes place over two periods, $t = 1, 2$. The utility of winning the contest is normalized to 1 and the utility of losing is equal to 0. In each stage, the agents' output gives rise to a stochastic score which indicates their relative performance. At the end of stage 2, the principal aggregates the scores from both stages to determine the winner of the contest. Agent A wins the contest if his accumulated output is greater than that of agent B , that is, if $x_1^A + x_2^A > x_1^B + x_2^B$ agent A wins and agent B wins if the reverse inequality holds.

Agent i's output in period t is given by $x_t^i = h(a^i, e_t^i) + \varepsilon_t^i$ where a^i is the agent's ability, e_t^i is the effort level, and ε_t^i is an error term. Each agent's ability a^i is identically and independently distributed and is initially unknown to the two agents and the principal. We assume that all players are Bayesian rational and have the same and correct prior beliefs about the distribution of ability. We further assume that these beliefs are common knowledge. Each agent's effort e_t^i is his private information and observed by neither the principal nor the other agent. Define the noise

4. Although not considering interim performance appraisals *per se*, Fang and Moscarini (2005) also discuss sorting and motivation effects that result from the informational content of wage-setting policies.

difference $\Delta\varepsilon_t \equiv \varepsilon_t^A - \varepsilon_t^B$ and assume that $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ are distributed independently and symmetrically around 0. Furthermore, we assume that $\Delta\varepsilon_t$ is distributed according to the cumulative density function $F(\cdot)$ and the density $f(\cdot)$ which is unimodal at 0 and twice continuously differentiable.

When exerting effort in period t agent i incurs a cost $c(e_t^i)$. We assume that $c(\cdot)$ is convex, $c'(0) = 0$ and $\lim_{e\to\infty} c(e) = \infty$. For simplicity, we shall assume that first-order conditions are sufficient to characterize optima.[5] The payoffs to agent i are

$$U^i = \Pr\left(x_1^i + x_2^i > x_1^j + x_2^j\right) - c\left(e_1^i\right) - c\left(e_2^i\right).$$

After the first stage, the principal privately observes the *difference* in first-period outputs between the two agents given by $\Delta x_1 \equiv x_1^A - x_1^B$. We assume that the principal wishes to maximize the sum of outputs of the contestants and that he can decide whether or not to credibly and publicly reveal this information to both players before the start of period 2. There are many situations in which it is natural to assume that feedback takes the form of such relative performance evaluation rather than agents receiving information about their absolute performance such as when output measures are subjective. For example, Moore and Klein (2008) report that both absolute and comparative performance feedback are frequently used in organizations. We further assume that the principal is committed to her feedback policy for any realization of the first-period output difference. This simplification allows us to suppress considerations of incentive problems on behalf of the principal such as misreporting the first-period output difference to elicit higher efforts or to selectively announce interim information.[6] The first-period output difference is favorable to agent A when $\Delta x_1 > 0$, favorable to agent B when $\Delta x_1 < 0$, and neutral or balanced when $\Delta x_1 = 0$. In the no-feedback scenario, neither of the agents knows first-period output when choosing second-period effort. In the feedback scenario, the principal is assumed to provide a truthful interim performance evaluation, so each agent learns the first-period outcome before choosing second-period efforts. We denote equilibrium efforts and outputs by e^* and x^* if the principal chooses a no-feedback policy and by $\widetilde{e}$ and $\widetilde{x}$ if she chooses to reveal information.

5. However, this is not true in general as has been frequently noted in the tournament literature. For a more detailed discussion of this point, see, for example, Nalebuff and Stiglitz (1983), O'Keefe et al. (1984), and Meyer (1992).

6. The assumptions of relative performance feedback and of commitment to a feedback policy are primarily made for reasons of tractability. Section 3.3 discusses the implications of relaxing these assumptions.

3. Interim Performance Evaluation without the Motivation Effect

In order to build intuition, we consider a special case of the general model in which the motivation effect is absent. This is the case in a model in which there is no heterogeneity in ability or when ability enters additively in the production function, that is, $x_t^i = a^i + e_t^i + \varepsilon_t^i$. If ability enters additively in the production function, learning plays no role in the agent's effort choice because the marginal benefit of effort is unaffected. By introducing heterogeneity in ability, we allow for time-persistent differences in relative outputs. When information is revealed, agents learn about relative first-period performance Δx_1 and update their beliefs about the ability difference between the two agents $E\left[\Delta a \mid \Delta x_1\right]$ where $\Delta a \equiv a^A - a^B$ which given the prior distributional assumptions is symmetrically distributed around 0.

3.1 No Feedback

If the principal chooses not to give feedback, then each agent only knows the effort he chose in the first period and so each agent's information is exactly the same in both periods. Thus, the problem is strategically equivalent to one where the first- and second-period efforts are chosen simultaneously at the beginning of stage 1. Agent A chooses first- and second-period efforts e_1^A and e_2^A to maximize

$$\begin{aligned} U^A &= \Pr\left(x_1^A + x_2^A > x_1^B + x_2^B\right) - c\left(e_1^A\right) - c\left(e_2^A\right) \\ &= E_{a^A,a^B,\Delta\varepsilon_1}\left[F\left(a^A + e_1^A + a^A + e_2^A - a^B - e_1^B - a^B - e_2^B + \Delta\varepsilon_1\right)\right] \\ &\quad - c\left(e_1^A\right) - c\left(e_2^A\right). \end{aligned}$$

The first-order condition yields

$$c'\left(e_t^A\right) = E_{a^A,a^B,\Delta\varepsilon_1}\left[f\left(2\Delta a + e_1^A + e_2^A - e_1^B - e_2^B + \Delta\varepsilon_1\right)\right]$$

for $t = 1, 2$.[7] Because the density function of the noise difference $f(\cdot)$ is symmetric around 0 and ability levels a^i are identically independently distributed, the expected marginal return to effort is the same for agent A and agent B. Given our assumptions, there is a unique interior solution to the first-order conditions for A and B, where $e^* \equiv e_t^{*i}$ for $i = A, B$ and $t = 1, 2$ solves the following first-order condition

$$c'(e^*) = E_{a^A,a^B,\Delta\varepsilon_1}\left[f(2\Delta a + \Delta\varepsilon_1)\right]. \qquad (1)$$

7. In what follows the first-order conditions for the two agents will be symmetric and so we omit writing them out for agent B.

3.2 Feedback

If the principal chooses to reveal information then the agents observe the gap in performance Δx_1 after the first period and take this into account when making their choice about second-period effort. We first solve for second-period effort levels for agent i for a given realization of Δx_1 and then solve for the optimal first-period efforts.

3.2.1 Second-Period Effort

In the second period, agent A solves the following maximization problem

$$\max_{e_2^A} E_{a^A,a^B}\left[F\left(\Delta x_1 + a^A + e_2^A - a^B - e_2^B\right) \mid \Delta x_1\right] - c\left(e_2^A\right).$$

The best-response effort choice in the second period is a function of the difference in first-period outputs Δx_1 as well as each agent's conjecture about the effort chosen by the opposing player in the first period. As we shall see in the analysis of the first period, agents will exert the same effort $\widetilde{e}_1$ in the first period. The symmetric second-period equilibrium, $\widetilde{e}_2(\Delta x_1) \equiv \widetilde{e}_2^i(\Delta x_1)$ for $i = A, B$ is unique and $\widetilde{e}_2(\Delta x_1)$ solves

$$c'(\widetilde{e}_2(\Delta x_1)) = E_{a^A,a^B}\left[f(\Delta x_1 + \Delta a) \mid \Delta x_1\right]. \tag{2}$$

From equation (2) we can see that revealing the first-period output difference Δx_1 has two effects. First, it informs contestants how large the difference is that favors one of the agents in the second period. Second, the revelation of Δx_1 also provides contestants with information about their relative ability difference Δa.

Consider first the case when there is no heterogeneity in ability so that Δa is always zero. In this case Δx_1 only reveals information about the gap that the "interim loser" of the tournament has to overcome in order to win. Second-period equilibrium effort $\widetilde{e}_2(\Delta x_1)$ is greatest when $\Delta x_1 = 0$ as the density $f(\cdot)$ reaches its maximum at this point. Moreover, due to the symmetry of the density of the noise difference around 0, second-period equilibrium effort is also symmetric around zero, that is, $\widetilde{e}_2(-\Delta x_1) = \widetilde{e}_2(\Delta x_1)$ and decreasing in the absolute value of the first-period output difference $|\Delta x_1|$. This means that equilibrium effort is lower the less balanced the second-period contest is between the two agents. This is the well-known *evaluation effect* or *lack-of-competition effect* that has received much attention in the literature on asymmetric tournaments (Schotter and Weigelt, 1992). Surprisingly, adding heterogeneity in ability to the model does not change these qualitative predictions given the symmetry assumptions we made about the distribution of ability. However, as we discuss below the quantitative effects of a revelation policy are more pronounced when agents also use the feedback given to them to update their beliefs about relative ability levels.

Because revealing information about the first-period output difference also reduces the noise that is present when agents make their choice about second-period output it is immediately clear that for sufficiently small values of $|\Delta x_1|$ second-period effort under a full-feedback policy is higher than second-period effort when no information is revealed. We call this effect the *noise reduction* effect of a full-feedback policy.

3.2.2 First-Period Effort

In the first period, agent A's maximization problem is

$$\max_{e_1^A} E_{a^A,a^B,\Delta\varepsilon_1}\left[F\left(2\Delta a + e_1^A + \tilde{e}_2^A - e_1^B - \tilde{e}_2^B + \Delta\varepsilon_1\right)\right]$$
$$-c\left(e_1^A\right) - E_{a^A,a^B,\Delta\varepsilon_1}\left[c\left(\tilde{e}_2^A\right)\right].$$

To obtain the first-order conditions we differentiate with respect to e_1^A. The first-order conditions are

$$\begin{aligned} c'\left(e_1^A\right) &= E_{a^A,a^B,\Delta\varepsilon_1}\left[f\left(2\Delta a + e_1^A + \tilde{e}_2^A - e_1^B - \tilde{e}_2^B + \Delta\varepsilon_1\right)\right] \\ &\quad + E_{a^A,a^B,\Delta\varepsilon_1}\left[\left\{f\left(2\Delta a + e_1^A + \tilde{e}_2^A - e_1^B - \tilde{e}_2^B + \Delta\varepsilon_1\right) - c'\left(\tilde{e}_2^A\right)\right\}\frac{d\tilde{e}_2^A}{de_1^A}\right] \\ &\quad - E_{a^A,a^B,\Delta\varepsilon_1}\left[f\left(2\Delta a + e_1^A + \tilde{e}_2^A - e_1^B - \tilde{e}_2^B + \Delta\varepsilon_1\right)\frac{d\tilde{e}_2^B}{de_1^A}\right]. \end{aligned} \tag{3}$$

The right-hand side of equation (3) is the marginal benefit of first-period effort which is composed of three parts. The first line captures the direct effect of first-period effort on the probability of winning the tournament whereas the second and third line capture the indirect effects on the contestant's own second-period effort $\tilde{e}_2^A$ and on the other contestant's second-period effort $\tilde{e}_2^B$. The following Lemma shows that in equilibrium only the direct effect will be present.

Lemma 1: *The unique first-period effort level under a feedback policy is given by*

$$c'(\tilde{e}_1) = E_{a^A,a^B,\Delta\varepsilon_1}\left[f(2\Delta a + \Delta\varepsilon_1)\right]. \tag{4}$$

Proof. See Appendix.

Lemma 1 demonstrates that in a setting where there is no heterogeneity in ability or when ability enters additively in the production function the contestants do not have any strategic incentive in the first period to exert effort to influence the second-period effort choice of his opponent. The key insight to show that the strategic effect is zero is the use of the symmetry property of the noise and ability difference.

3.3 Discussion

We begin the discussion by comparing effort and output levels under the two possible feedback policies.

Proposition 1: *First-period efforts are the same regardless of whether information is revealed or not, if the density of the sum of the per-period noise and the ability difference is symmetric around 0. The expected effort in the second period when information is revealed is lower (higher) than when information is not revealed if c' is convex (concave). Expected second-period effort (and overall effort) are the same under the two feedback policies if the cost function is quadratic.*

Proof. Using Lemma 1 we can compare equation (1) and (4) to show that $e^* = \widetilde{e}_1$. This completes the first part of the proof. Note further that because $\widetilde{e}_1 \equiv \widetilde{e}_1^A = \widetilde{e}_1^B$ we have $\Delta x_1 = \Delta a + \Delta\varepsilon_1$. Hence, equation (1) can be rewritten in the following form

$$\begin{aligned} c'(e^*) &= E_{a^A,a^B,\Delta\varepsilon_1}[f(2\Delta a + \Delta\varepsilon_1)] \\ &= E_{a^A,a^B,\Delta\varepsilon_1}[f(\Delta x_1 + \Delta a)] = E_{a^A,a^B,\Delta\varepsilon_1}[c'(\widetilde{e}_2(\Delta x_1))], \end{aligned}$$

which follows from equation (2) and the law of iterated expectations. Using Jensen's inequality we can provide a ranking of $\widetilde{e}_2(\Delta x_1)$ which is a random variable that depends on the realization of Δx_1 and e^* which is a constant. If c' is convex, Jensen's inequality implies

$$E[c'(\widetilde{e}_2(\Delta x_1))] \geq c'(E[\widetilde{e}_2(\Delta x_1)])$$

and because c' is increasing we therefore have

$$e^* \geq E[\widetilde{e}_2(\Delta x_1)].$$

Clearly, if c' is concave the reverse inequality holds. If the cost function is quadratic, then marginal cost is linear and expected second-period effort is the same regardless of whether information is revealed or not. □

As illustrated by the analysis, revealing information about relative performance has two competing effects on effort choice which we termed *noise reduction* and *evaluation effect*. On the one hand, revealing information reduces the overall noise component of output in the second period and thus increases incentives for second-period effort relative to the no-feedback scenario. On the other hand, revealing information about first-period performance invariably creates asymmetries between the agents and therefore tends to decrease second-period equilibrium efforts. If one agent has an advantage over the other, the contest will be less close and both agents will exert less effort. In our model, this phenomenon is reflected in the fact that second-period equilibrium effort $\widetilde{e}_2(\Delta x_1)$ is decreasing in the absolute value of first-period performance difference $|\Delta x_1|$. As a result, for realizations of Δx_1 that are close to 0, second-period effort will be higher under a full-feedback policy than under a no-feedback policy, whereas the opposite will be the case when Δx_1 is large in absolute terms. Whether the first or the latter effect prevails in expectation depends, as shown above, on the shape of the marginal cost function. As Proposition 1 shows, first-period effort is not altered by the revelation of information due to the absence of a strategic effect. In the examined model, under a full-feedback policy agents do not have an incentive to exert larger effort in the first period in order to carve out a leading position for the second period.[8]

We also showed that the same conclusions hold regardless of whether or not the agent's production function includes an additive ability term. The intuition for this result is the following. In a model without ability under a feedback policy second-period equilibrium effort depends on Δx_1 whereas in a model with additive ability second-period effort depends on Δx_1 *and* the posterior of Δa given Δx_1. Qualitatively, this means that nothing changes when we move from a model without ability to a model with additive ability. However, the quantitative effects of a full-feedback policy are more pronounced in a model with additive ability than in a model without ability because the ability difference and the first-period output difference are positively correlated. In particular, we have

$$\text{cov}(\Delta a, \Delta x_1) = 2\text{var}(a^i) > 0.$$

Not only does a gap in relative first-period performance create an uneven playing field between the contestants as one of the players has to overcome an output deficit of $\Delta x_1 > 0$, but given symmetric first-period effort choices the "interim loser" also expects to be disadvantaged with respect to ability in the second period. Furthermore, when effort costs are quadratic the expected (second-period) effort levels will be identical when comparing two tournaments that have the same amount

8. In the macroeconomics literature, there is an interesting analogue of the fact that the third derivative of the cost function plays a role in determining the relationship between the expected effort under the two possible feedback policies. In that literature, the combination of a positive third derivative of the utility function and uncertainty about future income reduces current consumption, and thus raises saving. This is known as *precautionary saving* (Leland, 1968). Similarly, in the interim evaluation setting the agent will exert *precautionary effort* in the second period if the marginal cost function is concave.

of aggregate uncertainty about the exogenous noise in output $\Delta\varepsilon_t$ and the persistent ability levels a^i but differ in how much uncertainty there is in each of those two components. However, actual second-period effort under a full-feedback policy will respond more strongly to the first-period output difference in the tournament that has more uncertainty about a^i and less uncertainty about $\Delta\varepsilon_t$.

The predictions of our baseline model change slightly when some of the more restrictive assumptions about commitment on behalf of the principal or the set of admissible feedback policies are changed. First, if the principal is able to costlessly manipulate the interim performance measure Δx_1 and report whichever outcome he desires, then the effort levels chosen by the contestants will be the same under no- and full-feedback policies. This is because the principal has an incentive to report the first-period output difference that maximizes the second-period efforts of the contestants in the second period regardless of the actual first-period output realization. Because the optimal message does not depend on the actual realization, only a babbling equilibrium can exist.[9] Second, if the principal is only able to selectively disclose interim information, that is, he can freely choose after the conclusion of the first period whether to announce or to withhold Δx_1, there will be full information unraveling (Grossman, 1981; Milgrom, 1981). Thus, in equilibrium, all information will be revealed and the contestants will choose the same effort levels as under a full-feedback policy.[10] Third, even if the principal is able to commit to a partial disclosure policy at the beginning of the relationship, Aoyagi (2007) shows that hybrid policies, which reveal the output difference Δx_1 only for a range of values but not for others, are never optimal as long as the marginal cost of efforts function is either convex or concave.[11] Finally, the choice between a full- and a no-feedback policy remains unchanged when we relax the assumption that the principal observes and reports the first-period output difference Δx_1. Just as in the present case where Δx_1 is revealed, when the agents receive information about both x_1^A and x_1^B, they will adjust their second-period effort choice to reflect this

9. For a detailed discussion of this point, see Ederer and Fehr (2007).

10. The full unraveling result crucially rests on the assumption that the informed party has perfect information concerning the payoff-relevant state, so that the pair of strategies in which the informed party announces the most favorable report consistent with the true state and the recipient of the report "assumes the worst," given an announcement, constitutes an equilibrium. Jung and Kwon (1988) and Shin (1994) point out that if there is some positive probability that a sender is unable to make a disclosure, either from prohibitively high disclosure costs or simple lack of information, then the unraveling result partially unravels. The analysis of such a model, however, is not straightforward and beyond the remit of this paper.

11. Note that this result is in contrast to Goltsman and Mukherjee (2008) who in a related but slightly different tournament setting find that partial disclosure can be optimal.

information. Yet the only information that is relevant for the agents' second-period effort choice is the difference in first-period efforts. As a result, the same argument based on Jensen's inequality will apply. However, if the principal can privately communicate x_1^A and x_1^B to the agents the optimal policy choice may be different and need not hinge exclusively on the curvature of the marginal cost function. As we shall demonstrate, the finding that the advantages of revealing information depend entirely on the sign of the third derivative of the cost function, also no longer applies when learning about ability impacts effort choices.

4. Interim Performance Evaluation with the Motivation Effect

To analyze the effects of motivation we employ a different formulation for the agent's production function which allows for complementarity between effort and ability. The simplest case that incorporates this feature is a situation where the production function takes the multiplicative form, $x_t^i = a^i e_t^i + \varepsilon_t^i$. When receiving interim feedback, as in the previous model, agents also learn about their ability. However, in contrast to the previous model, ability now directly influences the marginal benefit of effort.

The analysis proceeds in two steps. First, we investigate the motivation and signal-jamming effects of a full-feedback policy in isolation by assuming that each period's exogenous noise difference $\Delta\varepsilon_t$ is uniformly distributed. As we will argue below, under this distributional assumption the evaluation effect is not present. Second, to illustrate how the lack of competition between the two agents dampens the beneficial effects of a full-feedback policy, we assume that ability and the exogenous noise differences are normally distributed. We will assume that effort cost is quadratic, that is, $c(e_t^i) = \frac{k}{2}(e_t^i)^2,\ k > 0$. Note that under this assumption expected efforts and outputs are the same under both no-feedback and full-feedback policies in the previous model where no motivation effect was present.

4.1 The Uniform Model

We rule out the evaluation effect by assuming that $\Delta\varepsilon_1$ is uniformly distributed on the interval $[-d, d]$ and $\Delta\varepsilon_2$ is uniformly distributed between $[-m, m]$ where $m > d$.[12] As a result, the density of the sum

12. To see how this distributional assumption eliminates the evaluation effect, consider the case where there is no heterogeneity in ability (i.e., $\Delta a = 0$) and so the interim performance appraisal only contains information about Δx_1. When the noise

of first- and second-period error differences has a trapezoid shape and is symmetric around 0. Furthermore, for technical reasons we assume that a^i now has limited non-negative support $[0, \overline{a}]$, where $\overline{a} > 0$ has a log-concave density and a mean μ.

4.1.1 No Feedback

As in the previous section the problem faced by the agent in the case of a no-feedback policy of the principal is equivalent to one where the efforts are chosen simultaneously at the beginning of stage 1. Agent A's maximization problem is

$$\max_{e_1^A, e_2^A} E_{a^A, a^B, \Delta\varepsilon_1}\left[F\left(a^A e_1^A + a^A e_2^A - a^B e_1^B - a^B e_2^B + \Delta\varepsilon_1\right)\right] - c\left(e_1^A\right) - c\left(e_2^A\right).$$

Because ability is unknown throughout the two periods and the two agents have the same prior beliefs, by the same arguments as before there is a symmetric equilibrium where $e^* \equiv e_t^{*i}$ for $i = A, B$ and $t = 1, 2$ now solves

$$c'(e^*) = E_{a^i, a^j, \Delta\varepsilon_1}[f(2e^*\Delta a + \Delta\varepsilon_1)a^i] \quad \text{for } i \neq j.$$

This condition further simplifies once we note that $f(\cdot)$ is uniform

$$c'(e^*) = \frac{\mu}{2m}. \tag{5}$$

In contrast to the previous models, expected ability directly influences the effort choice of players. In this multiplicative output formulation, effort and ability are complements: higher expected ability implies a higher marginal benefit of effort and thus leads to a higher equilibrium effort level.[13]

4.1.2 Feedback

Second-period effort. In the second period, agent A solves

$$\max_{e_2^A} E_{a^A, a^B}\left[F\left(\Delta x_1 + a^A e_2^A - a^B e_2^B\right) \middle| \Delta x_1\right] - c\left(e_2^A\right).$$

The first-order condition for second-period effort is

$$c'\left(e_2^A\right) = E_{a^A, a^B}\left[f\left(\Delta x_1 + a^A e_2^A - a^B e_2^B\right) a^A \middle| \Delta x_1\right].$$

---

difference is uniformly distributed, effort is independent of Δx_1 because for a uniform density $f'(\cdot) = 0$ within its limited support. Equation (2) can be rewritten as

$$c'(\tilde{e}_2) = \frac{1}{2m}.$$

13. Note that throughout this section we focus exclusively on interior solutions for reasons of expositional clarity. In Appendix A.2, we derive sufficient conditions for the existence of interior solutions. These conditions require the cost function to be sufficiently convex and m to be sufficiently large relative to $\overline{a}$ and to the bounded support of $\Delta\varepsilon_1$.

Noting that the difference in noise is uniform, we obtain the first-order conditions for both agents which are given by

$$c'\left(\tilde{e}_2^A(\Delta x_1)\right) = \frac{1}{2m} E[a^A \mid \Delta x_1] \tag{6a}$$

$$c'\left(\tilde{e}_2^B(\Delta x_1)\right) = \frac{1}{2m} E[a^B \mid \Delta x_1]. \tag{6b}$$

The equilibrium effort in the second period is a function of Δx_1 and differs across agents. In particular, we have $\tilde{e}_2^A \neq \tilde{e}_2^B$ as long as $\Delta x_1 \neq 0$. This asymmetry in effort levels arises because the expected marginal benefit of effort will differ between the agents whenever the first-period output difference is unbalanced. Expected marginal benefit is higher for "winners" of the first-period contest (players for which Δx_1 is biased in their favor) and lower for first-period "losers." As a result, revelation of interim information will drive a wedge between the second-period equilibrium efforts of the two agents. We term this effect the *motivation effect* of a full-feedback policy.

First-period effort. In the first period, agent A's problem is

$$\max_{e_1^A} E_{a^A, a^B, \Delta\varepsilon_1}\left[F\left(a^A e_1^A + a^A \tilde{e}_2^A - a^B e_1^B - a^B \tilde{e}_2^B + \Delta\varepsilon_1\right)\right] \\ -c\left(e_1^A\right) - E_{\varepsilon_1^A, \varepsilon_1^B}\left[c\left(\tilde{e}_2^A\right)\right].$$

To obtain the first-order condition, we differentiate the above expression with respect to e_1^A. As before the first-order condition has three terms and is given by

$$\begin{aligned} c'\left(e_1^A\right) &= E_{a^A, a^B, \Delta\varepsilon_1}\left[f\left(a^A e_1^A + a^A \tilde{e}_2^A - a^B e_1^B - a^B \tilde{e}_2^B + \Delta\varepsilon_1\right) a^A\right] \\ &\quad + E_{a^A, a^B, \Delta\varepsilon_1}\left[\left\{f\left(a^A e_1^A + a^A \tilde{e}_2^A - a^B e_1^B - a^B \tilde{e}_2^B + \Delta\varepsilon_1\right) a^A - c'\left(\tilde{e}_2^A\right)\right\} \frac{d\tilde{e}_2^A}{de_1^A}\right] \\ &\quad - E_{a^A, a^B, \Delta\varepsilon_1}\left[f\left(a^A e_1^A + a^A \tilde{e}_2^A - a^B e_1^B - a^B \tilde{e}_2^B + \Delta\varepsilon_1\right) a^B \frac{d\tilde{e}_2^B}{de_1^A}\right]. \end{aligned} \tag{7}$$

Using the same line of reasoning as before, we show that the second line of the above equation is equal to zero by the envelope condition.

Lemma 2: *The first-order condition for first-period effort under a feedback policy is given by*

$$c'(\widetilde{e}_1) = \frac{1}{2m}\left\{\mu - E_{a^A,a^B,\Delta\varepsilon_1}\left[a^B \frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}\right]\right\}. \tag{8}$$

Proof. See Appendix.

4.1.3 Comparison and Discussion

Proposition 2: *If the distribution of the noise difference is uniform, then the first-period efforts and expected outputs are higher under a feedback policy than under a no-feedback policy.*

Proof. Comparing equations (5) and (8) immediately shows that they differ by the strategic effect that an increase in first-period effort of agent i has on the optimal second-period effort choice of agent j. To show that this strategic effect provides additional incentives for effort, we rewrite the second expression in the curly brackets of equation (8) in the following way

$$\begin{aligned}
&E_{a^A,a^B,\Delta\varepsilon_1}\left[a^B \frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}\right]\\
&\quad = E_{a^A,a^B,\Delta\varepsilon_1}\left[a^B \frac{d\widetilde{e}_2^B(\Delta x_1)}{d\Delta x_1}\frac{d\Delta x_1}{de_1^A}\right]\\
&\quad = \frac{1}{2m}E_{a^A,a^B,\Delta\varepsilon_1}\left[a^A a^B \frac{d}{d\Delta x_1}((c')^{-1}(E[a^B|\Delta x_1]))\right]\\
&\quad = \frac{1}{2m}E_{a^A,a^B,\Delta\varepsilon_1}\left[a^A a^B((c')^{-1})'\frac{d}{d\Delta x_1}(E[a^B|\Delta x_1])\right].
\end{aligned}$$

From the assumptions about the ability distribution, we know that all realizations of ability will be non-negative. Furthermore, because the cost function is convex marginal cost is increasing. Hence, the derivative of the inverse of the marginal cost function is positive. Because all expressions on the right-hand side are always non-negative with the exception of the derivative of the conditional expectation, we only need to show that this derivative is non-positive.

Efron (1965) has shown that $E[\theta(X_1, \ldots, X_n) \mid \sum X_i = w]$ is increasing in w for any increasing function θ when the X_i are independently distributed with log-concave densities. A special case of Efron's result is that $E[a^B \mid \widetilde{e}_1 a^A - \widetilde{e}_1 a^B + \Delta\varepsilon_1 = \Delta x_1]$ is decreasing in Δx_1 whenever $\widetilde{e}_1$ is a constant and a^A, a^B and $\Delta\varepsilon_1$ are independently distributed with log-concave densities.[14] Therefore, $\frac{d}{d\Delta x_1}(E[a^B \mid \Delta x_1]) \leq 0$ and so $\widetilde{e}_1 \geq e^*$.

As a result, expected combined first-period output is also higher under a feedback policy because

$$E_{a^A, a^B, \Delta\varepsilon_1}\left[\widetilde{x}_1^A + \widetilde{x}_1^B\right] = 2\mu\widetilde{e}_1 \geq 2\mu e^* = E_{a^A, a^B, \Delta\varepsilon_1}\left[x_1^{*A} + x_1^{*B}\right],$$

where the inequality follows from $\widetilde{e}_1 \geq e^*$. □

The result that effort is higher when feedback is provided is driven by the presence of implicit incentives that are similar in spirit to Holmstrom (1982) and Fudenberg and Tirole (1986) and which are absent when no feedback is given to the agents. We call this the *signal-jamming* or *strategic effect* of a full-feedback policy because it creates *implicit incentives* prior to the revelation of information to the agents. As discussed in the analysis of the effort choice in the second period, agents need to make conjectures about first-period effort choice of their opponent to update expectations of their own and their opponent's ability. As long as ability is unknown, each agent has an additional incentive to supply effort for any given conjecture of their opponent about first-period effort choice. By increasing effort, each agent can potentially bias the process of inference of his opponent in his favor. When player i increases his supply of effort, the first-period output difference will tend to move in i's favor which in turn will lead to a more pessimistic perception by agent j of his own ability at the start of period 2. Because effort and expected ability are complements, agent j will reduce his second-period effort, thereby increasing player i's probability of winning the tournament. Of course, in equilibrium neither agent will be able to fool his counterpart because the agents will know what effort levels to expect in equilibrium and adjust their beliefs accordingly.

Proposition 3: *In equilibrium, the expected effort in the second period is the same in both full-feedback and no-feedback scenarios if the distribution of the noise difference is uniform and the cost function is quadratic. The expected sum of second-period outputs of the two agents is higher when information is revealed than under a no-feedback policy.*

Proof. We rewrite equation (5) for quadratic effort cost and use the law of iterated expectations and equations (6a) and (6b)

$$e^* = \frac{\mu}{2mk} = E_{\Delta x_1}\frac{1}{2mk}E[a^i|\Delta x_1] = E_{\Delta x_1}\left[\widetilde{e}_2^i(\Delta x_1)\right],$$

14. Jewitt (1985) provides a discussion and generalization of Efron's (1965) result.

which shows that the expected second-period effort levels under both policies are equal.

The expected second-period output under a no-feedback policy is

$$E\left[x_2^{*A} + x_2^{*B}\right] = E\left[a^A e^* + \varepsilon_2^A + a^B e^* + \varepsilon_2^B\right] = \frac{\mu^2}{mk}.$$

The expected second-period output when information is revealed is

$$E\left[\tilde{x}_2^A + \tilde{x}_2^B\right] = E\left[a^A \tilde{e}_2^A + \varepsilon_2^A + a^B \tilde{e}_2^B + \varepsilon_2^B\right] = \frac{S}{2mk},$$

where the term S is given by

$$\begin{aligned} S &= E[a^A E[a^A \mid \Delta x_1] + a^B E[a^B \mid \Delta x_1]] \\ &= E_{\Delta x_1} E[a^A E[a^A \mid \Delta x_1] \mid \Delta x_1] + E_{\Delta x_1} E[a^B E[a^B \mid \Delta x_1] \mid \Delta x_1]. \end{aligned}$$

Jensen's inequality implies

$$\begin{aligned} &E_{\Delta x_1}(E[a^A \mid \Delta x_1])^2 + E_{\Delta x_1}(E[a^B \mid \Delta x_1])^2 \geq (E_{\Delta x_1} E[a^A \mid \Delta x_1])^2 \\ &\quad + (E_{\Delta x_1} E[a^B \mid \Delta x_1])^2 \end{aligned}$$

$$\Leftrightarrow$$

$$S \geq 2\mu^2.$$

Therefore, we have $E[\tilde{x}_2^A + \tilde{x}_2^B] \geq E[x_2^{*A} + x_2^{*B}]$. □

Under a full-feedback policy, more able players will, on average, receive good news in the form of a first-period output difference that is tipped in their favor, thus leading them to put in higher second-period effort than if they had relied upon their prior expectation of ability. Less able players on the other hand will tend to put in lower second-period effort, balancing out the increase of their more able competitors. If the cost function is quadratic, these two effects will exactly cancel out each other as shown in the analysis of the previous section.

The beneficial results of a full-feedback policy on second-period output are a consequence of the *motivation effect*. This might seem slightly paradoxical because expected effort is the same under no- and full-feedback scenarios if the cost function is quadratic. However, although expected effort does not differ across the two scenarios, actual second-period effort will differ. The key insight is that a full-feedback policy has a beneficial sorting effect because, on average, it raises the second-period effort of more able agents while reducing the effort of less able contestants. Given the complementary form of the production function, this leads to higher expected output than if both able and unable agents supply the same amount of effort as is the case under a no-feedback policy.

COROLLARY 1: *If the distribution of the noise difference is uniform and the cost function is quadratic, a full-feedback policy leads to higher expected overall output than a no-feedback policy.*

Proof. The corollary is a straightforward result of Propositions 2 and 3. Because expected output in both periods is higher under a feedback policy, expected overall output must be higher under such a policy. □

The superiority of a full-feedback policy is caused by the implicit incentives that exist in the first period as well as the sorting induced by the motivation effect of the second period. The combination of these effects guarantees that a principal solely interested in output maximization will choose to implement an interim performance evaluation.

The most important difference that the complementarity between effort and ability generates is the asymmetric effect that feedback has for the second-period effort choice of the two agents. When effort and ability are complementary, "winners" of the first period will choose higher levels than "losers" because their expected marginal benefits of effort are no longer the same. "Winners" are more motivated and have more confidence in their own ability, that is to say they hold higher expectations about their own ability. Of course, this motivation effect is a double-edged sword because it also breaks bad news to some workers and depresses their morale and effort levels. However, by motivating and demotivating the right kinds of contestants, a full-feedback policy effectively sorts the workers. Proposition 3 shows that because, on average, a full-feedback policy transmits good news to more able and bad news to less able individuals, expected output is higher under a full-feedback policy than under a no-feedback rule.

It is straightforward to show that the advantages of this sorting effect are higher the stronger the agents' beliefs about ability respond to the first-period output difference. This can be seen from the difference in second-period output between the two feedback policies which is given by

$$E\left[\tilde{x}_2^A + \tilde{x}_2^B\right] - E\left[x_2^{*A} + x_2^{*B}\right] = \frac{1}{2mk}\operatorname{var}(E[a^i \mid \Delta x_1]),$$

where the variance is taken over all values of Δx_1. Hence, the more variable is the agents' posterior of ability following the revelation of Δx_1, the larger is the difference in output between the two policies. In the no-feedback scenario workers will still choose the same effort levels regardless of their actual abilities as effort choice only depends on expected ability. Under a full-feedback policy, agents choose asymmetric effort levels in the second period as they tailor their effort choice to their posterior about their ability level so that on average output will be higher. The favorable outcomes of this sorting effect will also be larger when the principal only cares about the effort of the most able individuals or when it is important to select the most able individual for promotion. This may be particularly important in industries where only a few select, highly able individuals advance up the corporate ladder. For example, professional service firms companies must pay particular attention to the quality of the few analysts who are promoted to the partner level (Levin and Tadelis, 2005), and indeed frequent and highly institutionalized interim performance appraisals are a common feature in such firms.

The asymmetries that arise from the motivation effect are also the source of the implicit incentives that are present in the first period of the full-feedback scenario. As before, if ability is uncertain, each agent puts some weight on Δx_1 when revisiting his beliefs about ability. If ability is multiplicative, an imbalance in the first-period output difference has direct repercussions on the second-period effort choice regardless of the opponent's reaction in the second period. Observing an unfavorable realization of Δx_1 leads to a lower effort choice because the agent has less confidence in his ability, and to a lower equilibrium effort choice because the second-period tournament is unequal. Whereas the second effect is also present in a model with no heterogeneity in ability or additive ability, the first effect is only a feature of the multiplicative ability model. Thus, both contestants have an incentive to influence their respective opponent's perceptions about ability by exerting additional effort that shifts Δx_1 in their favor.[15] However, if ability levels are commonly known, there are no returns to influencing output and the implicit incentives vanish.

We conclude this section with a note of caution. The derivation of the results of Section 4.1 relied on the assumption that the noise difference is uniformly distributed. This assumption eliminates the *evaluation effect*. As discussed in Section 3, this evaluation effect causes equilibrium effort levels to change with the closeness of competition between the two agents. The effect is absent in the uniform model as the density function and hence the marginal benefit of effort do not vary with the difference in first-period output because for a uniform density $f'(\cdot) = 0$ within its limited supports. Therefore, in the uniform model with multiplicative ability the asymmetries created by a full-feedback policy do not have any adverse effects on effort provision. Hence, although it would be tempting to conclude that within an exogenously given incentive scheme the principal will favor the use of

15. Note that this signal-jamming effect is similar to the career concerns in Holmstrom's (1982) model where implicit incentives are provided because the agent has an incentive to bias the market's assessment of his ability by supplying additional effort.

interim performance evaluation, such a conclusion would be premature. In the next section, we analyze how the use of a different distribution for the noise difference that allows for the presence of the evaluation effect weakens the argument for full-feedback policies.

4.2 The Normal Model

We now assume that the noise difference $\Delta\varepsilon_t$ is normally rather than uniformly distributed. This serves to illustrate the evaluation effect that will tend to weaken the results found in previous sections. In order to ensure expositional clarity, we assume that the cost function is quadratic and that all random variables are normally distributed. Let the prior joint distribution of random variables therefore be given by

$$\begin{pmatrix} a^A \\ a^B \\ \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \end{pmatrix} \sim N\left[\begin{pmatrix} \mu \\ \mu \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 2\tau^2 & 0 \\ 0 & 0 & 0 & 2\tau^2 \end{pmatrix}\right].$$

Rather than providing a complete comparison of both full-feedback and no-feedback scenarios we limit our attention to showing how the evaluation effect undermines the beneficial consequences of a full-feedback policy because, even with these simplifying distributional assumptions, the solutions are too algebraically complex to solve explicitly for effort levels. However, the distributional assumptions allow us to illustrate the tradeoff between the evaluation and the motivation effect. Moreover, we are able to choose parameter values for which the maximization problems of the agents are concave and we can numerically solve for (unique) equilibria in pure strategies. This enables us to contrast the effects on effort and output of the two different feedback policies as well as analyze the effect of changes in the parameter values on the relative strengths of the evaluation and the motivation effect as well as the sorting that the latter induces. Most importantly, the re-introduction of the evaluation effect and its interaction with the motivation effect destroys the optimality of a full-feedback policy that we obtained in the previous section. For different parameter values, one or the other feedback policy may dominate.

4.2.1 No Feedback

Given the joint normality of all variables, agent A's probability of winning the contest is

$$\Pr(\Delta x_1 + \Delta x_2 > 0) = \Phi(z^A),$$

where $z^A = \frac{E[\Delta x_1 + \Delta x_2]}{\sqrt{\text{var}\,(\Delta x_1 + \Delta x_2)}}$ and Φ denotes the standard normal cumulative distribution function. The first-order condition is given by

$$\phi(z^A)\frac{\partial z^A}{\partial e^A} = ke^A.$$

Again we note that equilibrium effort levels are the same for both agents in both periods, that is, $e_t^{i*} = e^*$ for all $i = A, B$ and $t = 1, 2$. The first-order condition therefore reduces to

$$ke^* = \frac{\phi(0)}{\sqrt{\text{var}\,(\Delta x_1 + \Delta x_2)}}\mu, \tag{9}$$

where ϕ is the standard normal probability density function. This solution has all the obvious interpretations of tournament models. Equilibrium effort is increasing in the expected level of ability μ and the degree of competition in the tournament $\phi(0)$ which is at its optimum at 0. Equilibrium effort is decreasing in the overall uncertainty of the contest given by var $(\Delta x_1 + \Delta x_2)$ which is increasing in the variance of ability and exogenous noise.

4.2.2 Feedback

Given a first-period output difference Δx_1, agent A's probability of winning the contest is given by

$$\Pr(\Delta x_1 + \Delta x_2 > 0 \mid \Delta x_1) = \Phi\left(y_2^A\right),$$

where $y_2^A = \frac{\Delta x_1 + E[\Delta x_2 | \Delta x_1]}{\sqrt{\text{var}(\Delta x_2 | \Delta x_1)}}$. The first-order conditions for the two players yield

$$\phi(y_2)\frac{\partial y_2^A}{\partial e_2^A} = ke_2^A,$$

$$\phi(y_2)\frac{\partial y_2^B}{\partial e_2^B} = ke_2^B$$

because $y_2^A = -y_2^B$ and hence $\phi(y_2) \equiv \phi(y_2^i)$ for $i = A, B$. These first-order conditions lead to asymmetric effort choices and their explicit form is derived in Appendix A.4. From the previous analysis, we know that there is a symmetric first-period equilibrium where the two *ex ante* symmetric players choose the same effort levels, that is, $\widetilde{e}_1 \equiv \widetilde{e}_1^i$ for $i = A, B$. Using the distributional assumptions we find that the first-order conditions are

$$ke_2^A = \frac{\phi(y_2)}{\sqrt{\text{var}(\Delta x_2 \mid \Delta x_1)}} \times \left\{ \mu + \frac{\sigma^2 e_1}{\text{var}(\Delta x_1)} \Delta x_1 - \frac{\sigma^2 \left[e_2^A - \frac{\text{cov}(\Delta x_1, \Delta x_2)}{\text{var}(\Delta x_1)} e_1 \right]}{\text{var}\,(\Delta x_2 \mid \Delta x_1)} T \right\} \tag{10a}$$

$$ke_2^B = \frac{\phi(y_2)}{\sqrt{\text{var}\,(\Delta x_2 \mid \Delta x_1)}} \times \left\{ \mu - \frac{\sigma^2 e_1}{\text{var}(\Delta x_1)} \Delta x_1 + \frac{\sigma^2 \left[e_2^B - \frac{\text{cov}(\Delta x_1, \Delta x_2)}{\text{var}(\Delta x_1)} e_1 \right]}{\text{var}\,(\Delta x_2 \mid \Delta x_1)} T \right\}, \tag{10b}$$

where $T = \Delta x_1 + E\,(\Delta x_2 \mid \Delta x_1)$.

From equations (10a) and (10b), one can immediately see that the asymmetry in effort levels is created by the interplay of two different factors: imbalances in the first-period output difference Δx_1 and variance in ability σ^2. These correspond to the evaluation and the motivation effect. First, note that when $\Delta x_1 = 0$, the effort choice of the agents is symmetric for any σ^2 , that is, $\widetilde{e}_2^A = \widetilde{e}_2^B$ and the above equations reduce to

$$k\widetilde{e}_2(\Delta x_1) = \frac{\phi(0)}{\sqrt{\text{var}\,(\Delta x_2 \mid \Delta x_1)}} \mu.$$

Second, when $\sigma^2 = 0$, the model reduces to the case where there is no motivation effect and the symmetric equilibrium effort level which is decreasing in Δx_1 is given by

$$k\widetilde{e}_2(\Delta x_1) = \frac{\phi\left(\frac{\Delta x_1}{\sqrt{\text{var}\,(\Delta x_2 | \Delta x_1)}}\right)}{\sqrt{\text{var}\;(\Delta x_2 \mid \Delta x_1)}} \mu.$$

Figure 1 illustrates how the motivation effect increases the asymmetry of second-period effort choices. For the chosen parameter values, the maximization problems for the two agents are concave and we can numerically compute their best response functions for different realizations of Δx_1. The best response functions when $\sigma^2 = 1$ are shown in Figure 1 for three different values of Δx_1. Note, in particular, that equilibrium effort choices are no longer symmetric as soon as $\Delta x_1 \neq 0$ which can be most clearly seen from the different unique intersection

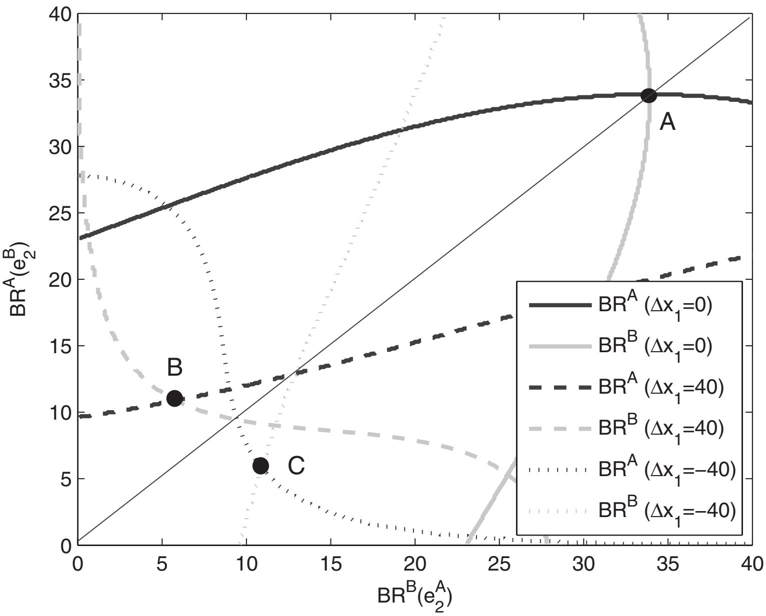

FIGURE 1. SECOND-PERIOD BEST RESPONSE FUNCTIONS UNDER A FULL FEEDBACK POLICY ($\mu = 1$, $\sigma^2 = 1$, $\tau^2 = 400$, *AND* $k = 0.00027$)

points of the best responses for $\Delta x_1 = 40$ (point B) and $\Delta x_1 = -40$ (point C).

Holding the overall noise in the tournament constant, that is, $\mathrm{var}\,(\Delta x_1 + \Delta x_2)$ is held constant so that the effort e^* under a no-feedback policy would be unchanged, we can also compute and plot the second-period equilibrium effort choices $\widetilde{e}_2^A(\Delta x_1)$ of agent A for given values of Δx_1 for different values of σ^2 which are shown in Figure 2. The equilibrium effort choices are entirely symmetric around 0 when $\sigma^2 = 0$, but they become more asymmetric as σ^2 increases (and exogenous noise τ^2 decreases). For example, for $\sigma^2 = 1$ agent A chooses $\widetilde{e}_2(\Delta x_1 = 40) \approx 12$ which is more than twice as large as $\widetilde{e}_2(\Delta x_1 = -40) \approx 5$. As argued before, this asymmetry is a result of the motivation effect because agent A (correctly) believes that he has a higher marginal benefit of effort when he is given favorable information than when he receives the corresponding negative information.

4.2.3 Comparison and Discussion

We decompose equations (10a) and (10b) into several parts and compare them with equation (9).

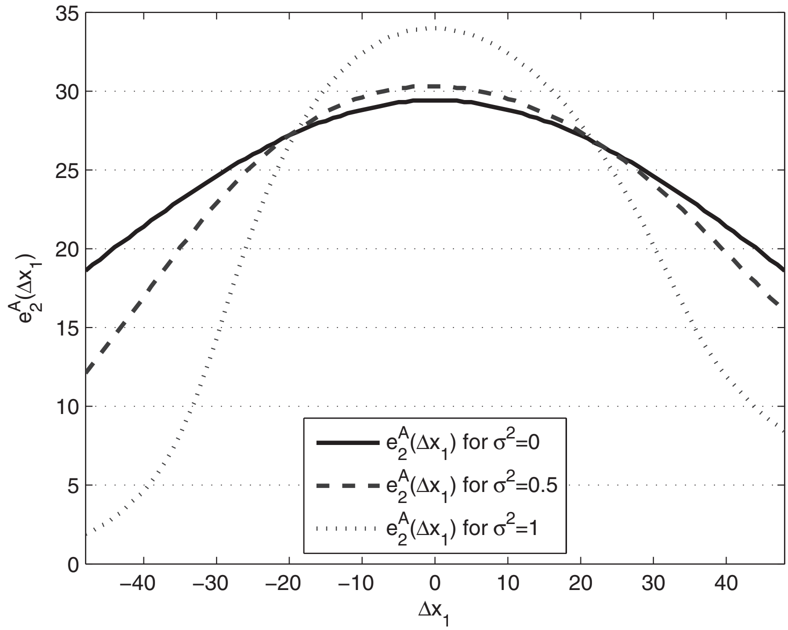

FIGURE 2. SECOND-PERIOD EQUILIBRIUM EFFORT CHOICE $\widetilde{e}_2^A(\Delta x_1)$ *OF PLAYER A UNDER A FULL FEEDBACK POLICY* (constant $\text{var}(\Delta x_1 + \Delta x_2)$, $\mu = 1, k = 0.00027$)

The first term on the right-hand outside the brackets resembles the first term on the right-hand side of equation (9) and captures the evaluation and risk reduction effects. The numerator $\phi(y_2)$ shows that the expected marginal benefit of second-period effort is higher the closer the contest is. When $\Delta x_1 = 0$, the second-period tournament is symmetric and $y_2 = 0$ so that $\phi(y_2)$ reaches its maximum. However, whenever the first-period output difference is biased in favor of one of the agents, that is, $\Delta x_1 \neq 0$, y_2 will be different from zero and $\phi(y_2) < \phi(0)$. The denominator is a measure of the noise of the second period. Information revelation reduces the amount of noise as can be seen from the relation $\text{var}(\Delta x_2 \mid \Delta x_1) < \text{var}(\Delta x_1 + \Delta x_2)$ and therefore increases incentives for effort.

Just as in the previous models, second-period effort is increasing in the prior expected ability of the agent. This can be seen from the first term inside the brackets on the right-hand side of (10a) and (10b). The second term inside the curly brackets of equation (10a) and (10b) captures the motivation effect. An observation of the first-period output difference that is favorable to player A, that is, $\Delta x_1 > 0$, increases the effort supplied by agent A and decreases that of agent B by the same amount because $\frac{\sigma^2 e_1}{\text{var}(\Delta x_1)} > 0$.

However, the asymmetry in second-period effort choices that is created by a realization of the first-period output difference other than zero through the third term which captures the interaction of motivation and evaluation effect. When the output difference Δx_1 is positive, agent A realizes that on average he is more able than agent B and that he can therefore produce the same amount of output as his opponent even when he exerts less effort. It is indeed possible that for some values of $\Delta x_1 > 0$ agent A actually exerts less effort in equilibrium than agent B . In such a situation, the beneficial sorting role of the motivation effect is actually reversed as less able agents exert more effort at least for some realizations of Δx_1.

Furthermore, the numerical analysis where we solve for the equilibrium first- and second-period effort levels under both feedback policies reveals that there is no longer an unambiguous ranking of feedback policies. It is now possible that either feedback policy may dominate the other. For example, for the different parameter values chosen for the numerical analysis in Figure 1, for $\sigma^2 = 1$ a full-feedback policy leads to lower expected second-period effort (evaluation effect) but higher first-period effort (signal-jamming effect) and higher expected second-period output (motivation and sorting effect) and therefore higher aggregate output than a no-feedback policy. On the other hand, for $\sigma^2 = 0.5$ both expected aggregate effort and output are lower under a full-feedback policy. Because we chose quadratic effort costs, the two policies are obviously identical when $\sigma^2 = 0$. Moreover, it is not true that a larger variance σ^2 generally favors a full-feedback policy because the asymmetries in second-period effort may become too large.

The analysis makes clear that in a setting where workers are *ex ante* identical, the asymmetries created by a full-feedback policy have disadvantageous consequences for effort provision when one considers distribution functions for the noise difference other than the uniform distribution. An immediate consequence of this argument is that for motivational purposes it is not always optimal to give workers full information about what the firm thinks of their performance. It is therefore not surprising that an important strand of research in organizational behavior argues that encouraging managers to be more open about performance information can be counterproductive (Meyer et al., 1965), especially when such information leads them to think that they are not close to the margin between winning and losing a promotion contest (Gibbs, 1991). Furthermore, some researchers advocate a system in which managers are not too open with employees, but actively use information to manage employee perceptions of performance and therefore motivation (Beer, 1987; Murphy and Cleveland, 1995).

Our analysis suggests a fundamental tradeoff that firms face when deciding whether or not to give feedback to their employees. On the one hand, giving feedback gives the firm the opportunity to motivate workers and to allow them to make more informed decision about how much effort to exert. Such a revelation policy also creates implicit incentives for agents to supply effort. On the other hand, performance appraisals create an uneven playing field between the agents that may lead to a lack of competition. This suggests a distinction between two types of information which are important in interim performance appraisals. The first is information about how the employee can improve performance and develop skill, whereas the second is information about a worker's future prospects. This distinction corresponds to two personnel objectives that, according to the human resource management literature, are often in conflict: "training and development" on the one hand and "evaluation" on the other.

In terms of the model we proposed, the first kind of information is information that helps the agent in tailoring effort to his correct ability level and the second is information about how likely an agent is to win tournament. Clearly, the first type of information has positive consequences for total output. In contrast, information of the second type creates an uneven playing field between the agents and is therefore unlikely to have favorable implications for effort choice. In theory, if an organization can separate these two forms of information, it will want to give the employee as much as possible of the former kind, while suppressing the latter kind. In practice, however, they are hard to separate, creating a tension that has received much attention in the organizational behavior literature. Beer (1987) notes that the two overall goals of "coaching and development" and "evaluation" are generally in conflict, and that a firm's approach to performance appraisal involves a balance of this tension. Gibbs (1991) documents that organizations make deliberate attempts to mitigate the conflict between evaluation and development. Some firms have separate appraisals at different times of the year, one for evaluation and one for development. In addition, many organizations explicitly state on assessment forms that they are not for the purpose of evaluation. For example, consulting firms perform periodic formal performance assessments in which entry-level analysts appraise the performance of the consultants and partners they work with and although helpful for the recipients in terms of development, these appraisals have little impact on their evaluation.

5. Concluding Remarks

This paper has studied the role of organizational feedback mechanisms. Its main contribution is the consideration of heterogeneity among the agents, a generalization that opens the way for the analysis of learning on behalf of the agents as well as of motivational measures taken by the firm. In models in which ability plays no role or enters additively, the latter aspect is absent and so the choice between a no-feedback and a full-feedback policy is shown to depend on the third derivative of the disutility of effort in a way that is reminiscent of the macroeconomic phenomenon of precautionary saving. However, the analysis becomes much richer in content once one considers a situation in which beliefs about ability directly impact each worker's effort choice. In such a setting, interim performance evaluations are not merely a mechanism by which information about past performance is transmitted to workers: They also represent an intentional attempt by organizations to influence the morale of their workers. We discuss several consequences of interim performance evaluations such as the signal-jamming and the motivation effect (as well as the efficient sorting of workers that the latter induces) that have not been analyzed in previous economic models, but are key to our understanding of performance appraisal processes in organizations.

However, because organizations are faced with several countervailing incentives, the model presented in this paper does not provide an unequivocal endorsement of interim performance evaluations. Interim performance evaluations motivate some employees, but at the same time the information they convey will demotivate other workers and may also reduce equilibrium effort of all workers, in particular, when this information creates a very uneven playing field between the contestants in future periods. Not only does this suggest a fundamental tradeoff with respect to performance appraisals that is in accordance with the organizational behavior literature, but it also indicates that there might be cross-industry variation in the prevalence of organizational feedback mechanisms. In particular, we would expect to find a higher percentage of organizations conducting interim performance evaluations in industries in which the motivation and sorting effects of such evaluations are particularly important. As pointed out in Section 4.1, this prediction matches the widespread use of interim performance appraisals in the professional service industries.

While shedding a first light on the previously uncharted territory of motivation our analysis can be extended in numerous ways. For example, contrary to our assumptions, interim performance appraisals sometimes take the form of private communication. Furthermore, there is ample evidence that interim performance appraisals are not always truthful, but are in fact manipulated by superiors to increase worker performance. Finally, our analysis focuses exclusively on maximizing output under the two scenarios when the incentive scheme is exogenously given, but we do not allow the principal to choose the incentive scheme optimally in the two scenarios.

Appendix A: Omitted Proofs and Calculations

A.1 Proof of Lemma 1

Proof. The second line of (3) is zero by the envelope theorem. Rewriting and using the law of iterated expectations yields

$$\begin{aligned}
&E_{\Delta x_1} E_{a^A, a^B, \Delta\varepsilon_1}\left[\left\{f(\Delta x_1 + \Delta a) - c'\left(\widetilde{e}_2^A(\Delta x_1)\right)\right\} \frac{d\widetilde{e}_2^A(\Delta x_1)}{de_1^A}\middle| \Delta x_1\right] \\
&= E_{\Delta x_1} \frac{d\widetilde{e}_2^A(\Delta x_1)}{de_1^A} E_{a^A, a^B, \Delta\varepsilon_1}\left[f(\Delta x_1 + \Delta a) - c'(\widetilde{e}_2^A(\Delta x_1))\middle| \Delta x_1\right] \\
&= E_{\Delta x_1} \frac{d\widetilde{e}_2^A(\Delta x_1)}{de_1^A} E_{a^A, a^B}\left[f(\Delta x_1 + \Delta a) - c'\left(\widetilde{e}_2^A(\Delta x_1)\right)\middle| \Delta x_1\right] \\
&= 0
\end{aligned}$$

by the first-order conditions of the second period.

We now show that in the additive ability model the strategic effect given by the third line of (3) is zero as well. First, we note that because the agents are *ex ante* symmetric both in terms of production functions as well as beliefs, there is a symmetric first-period equilibrium, that is, $\widetilde{e}_1^A = \widetilde{e}_1^B \equiv \widetilde{e}_1$. Hence we can rewrite the third line to read

$$\begin{aligned}
&E_{a^A, a^B, \Delta\varepsilon_1}\left[f(\Delta x_1 + \Delta a)\frac{d\widetilde{e}_2^B}{de_1^A}\right] \\
&= E_{\Delta x_1} E_{a^A, a^B, \Delta\varepsilon_1}\left[f(\Delta x_1 + \Delta a)\frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}\middle| \Delta x_1\right] \\
&= E_{\Delta x_1} E_{a^A, a^B}\left[f(\Delta x_1 + \Delta a)\frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}\middle| \Delta x_1\right] \\
&= E_{\Delta x_1} \frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A} E_{a^A, a^B}[f(\Delta x_1 + \Delta a) \mid \Delta x_1].
\end{aligned}$$

Substituting from (2) one obtains

$$E_{\Delta x_1}\left[\frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}c'\big(\widetilde{e}_2^A(\Delta x_1)\big)\right] = E_{\Delta x_1}\left[\frac{d\widetilde{e}_2^B(\Delta x_1)}{d\Delta x_1}c'\big(\widetilde{e}_2^A(\Delta x_1)\big)\right]$$
$$= 0$$

because $c'(\widetilde{e}_2^A(\Delta x_1))$ and $\frac{d\widetilde{e}_2^B(\Delta x_1)}{d\Delta x_1}$ are symmetric in Δx_1 around 0 because of the symmetry properties of $\widetilde{e}_2^A(\Delta x_1)$ and $\widetilde{e}_2^B(\Delta x_1)$.

Noting the symmetry of first- and second-period efforts the first-order condition therefore simplifies to

$$E_{a^A,a^B,\Delta\varepsilon_1}[f(2\Delta a + \Delta\varepsilon_1)] = c'(\widetilde{e}_1).$$

□

A.2 Sufficient Conditions for Interior Solutions

In Section 4.1, we assume that the noise difference is uniformly distributed. Because this means that the noise difference has limited support one must pay particular attention to corner solutions. In what follows we derive sufficient conditions for there to be no corner solutions.

A.2.1 No Feedback

Under a no-feedback policy, the first-order condition is given by

$$c'(e^*) = E_{a^i,a^j,\Delta\varepsilon_1}[f(2e^*\Delta a + \Delta\varepsilon_1)a^i].$$

If we assume that

$$-m \leq 2e^*\Delta a + \Delta\varepsilon_1 \leq m \tag{A1}$$

holds for all possible realizations of Δa and $\Delta\varepsilon_1$, we can rewrite the first-order condition in this way

$$\frac{\mu}{2m} = c'(e^*)$$

as we have done in the exposition of the uniform model in Section 4.1. Equation (A1) holds if

$$2e^*\overline{a} + 2d \leq m.$$

Here, we have substituted the most extreme realizations of Δa and $\Delta\varepsilon_1$. Finally substituting in for e^* we have

$$2\overline{a}\left(\frac{\mu}{2mk}\right) + 2d \leq m,$$

which holds if the noise in period 2, m, and/or the scale of the marginal cost of effort k are sufficiently large.

A.2.2 Feedback

Following the same procedure as before, we now have to ensure that the following condition holds:

$$-m \leq \widetilde{e}_1 \Delta a + a^A \widetilde{e}_2^A(\Delta x_1) - a^B \widetilde{e}_2^B(\Delta x_1) + \Delta\varepsilon_1 \leq m.$$

Substituting the most extreme values for a^i and $\Delta\varepsilon_1$, as well as $\widetilde{e}_2^i$ we obtain

$$\overline{a}\left\{\widetilde{e}_1 + \frac{1}{2mk} E[a^A \mid \Delta x_1]\right\} + 2d \leq m,$$

where $\widetilde{e}_1 = \frac{1}{2mk}\{\mu - E_{a^A,a^B,\Delta\varepsilon_1}[a^B \frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}]\}$. Again, this inequality holds if m and/or the scale of the marginal cost of effort k are sufficiently large.

Comparing the conditions for the no-feedback and full-feedback case shows that because of the asymmetries of the full-feedback scenario the sufficient conditions derived for the revelation case are stricter. We therefore assume that throughout the analysis these conditions hold, an assumption which allows us to rule out unwanted corner solutions.

A.3 Proof of Lemma 2

Proof. The second line of (7) is zero by the envelope theorem. Using the law of iterated expectations yields

$$\begin{aligned}
&E_{\Delta x_1} E_{a^A,a^B,\Delta\varepsilon_1}\left[\left\{ f\left(\Delta x_1 + a^A \widetilde{e}_2^A(\Delta x_1) - a^B \widetilde{e}_2^B(\Delta x_1)\right) a^A \right.\right.\\
&\quad \left.\left. - c'\left(\widetilde{e}_2^A(\Delta x_1)\right)\right\} \frac{d\widetilde{e}_2^A(\Delta x_1)}{de_1^A} \right| \left. \Delta x_1 \right]\\
&\quad = E_{\Delta x_1} \frac{d\widetilde{e}_2^A(\Delta x_1)}{de_1^A} E_{a^A,a^B,\Delta\varepsilon_1}\left[\left\{ f\left(\Delta x_1 + a^A \widetilde{e}_2^A(\Delta x_1) - a^B \widetilde{e}_2^B(\Delta x_1)\right) a^A \right.\right.\\
&\qquad \left.\left. - c'\left(\widetilde{e}_2^A(\Delta x_1)\right)\right\} \right| \left. \Delta x_1 \right]\\
&\quad = E_{\Delta x_1} \frac{d\widetilde{e}_2^A(\Delta x_1)}{de_1^A} E_{a^A,a^B}\left[\left\{ f\left(\Delta x_1 + a^A \widetilde{e}_2^A(\Delta x_1) - a^B \widetilde{e}_2^B(\Delta x_1)\right) a^A \right.\right.\\
&\qquad \left.\left. - c'\left(\widetilde{e}_2^A(\Delta x_1)\right)\right\} \right| \left. \Delta x_1 \right]\\
&\quad = 0,
\end{aligned}$$

where the last equality again follows from the first-order condition of the second period.

Because the agents are *ex ante* identical in terms of production function and beliefs there is a symmetric equilibrium of the first period where $\widetilde{e}_1^A = \widetilde{e}_1^B \equiv \widetilde{e}_1$ so that the first-order conditions simplify to

$$E_{a^A,a^B,\Delta\varepsilon_1}\left[f\left(\widetilde{e}_1\Delta a + a^A\widetilde{e}_2^A(\Delta x_1) - a^B\widetilde{e}_2^B(\Delta x_1) + \Delta\varepsilon_1\right)\left(a^A - a^B\frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}\right)\right]$$
$$= c'(\widetilde{e}_1).$$

Using the assumptions about the density function, we obtain the more tractable condition

$$\frac{1}{2m}\left\{\mu - E_{a^A,a^B,\Delta\varepsilon_1}\left[a^B\frac{d\widetilde{e}_2^B(\Delta x_1)}{de_1^A}\right]\right\} = c'(\widetilde{e}_1).$$

□

A.4 Calculations for the Normal Model

A.4.1 No Feedback

The first-order conditions under a no-feedback policy are given by

$$\phi(z^A)\frac{\partial z^A}{\partial e^A} = ke^A.$$

Furthermore, we have

$$\Delta x_1 + \Delta x_2 \sim N\left(\mu\left(e_1^A - e_1^B + e_2^A - e_2^B\right), \sigma^2\left[\left(e_1^A + e_2^A\right)^2 + \left(e_1^B + e_2^B\right)^2\right] + 4\tau^2\right).$$

The second part of the left-hand side of the first-order condition therefore yields

$$\frac{\partial z^A}{\partial e^A} = \frac{1}{\sqrt{\mathrm{var}\,(\Delta x_1 + \Delta x_2)}}\left[\mu - \frac{\mu\left(e_1^A - e_1^B + e_2^A - e_2^B\right)\sigma^2\left(e_1^A + e_2^A\right)}{\mathrm{var}\,(\Delta x_1 + \Delta x_2)}\right]$$

so that at the symmetric equilibrium the first-order condition reduces to

$$ke^* = \frac{\phi(0)}{\sqrt{\mathrm{var}\,(\Delta x_1 + \Delta x_2)}}\mu.$$

A.4.2 Feedback

The first-order conditions for the two players yield

$$\phi(y_2)\frac{\partial y_2^A}{\partial e_2^A} = ke_2^A$$

$$\phi(y_2)\frac{\partial y_2^B}{\partial e_2^B} = ke_2^B.$$

Noting that Δx_t is normally distributed for given levels of e_t^i allows us to write

$$\begin{aligned}
\text{var}(\Delta x_1) &= \left[\left(e_1^A\right)^2 + \left(e_1^B\right)^2\right]\sigma^2 + 2\tau^2 \\
\text{var}(\Delta x_2) &= \left[\left(e_2^A\right)^2 + \left(e_2^B\right)^2\right]\sigma^2 + 2\tau^2 \\
\text{cov}(\Delta x_1, \Delta x_2) &= \left(e_1^A e_2^A + e_1^B e_2^B\right)\sigma^2 \\
E(\Delta x_2 \mid \Delta x_1) &= \mu\left(e_2^A - e_2^B\right) + \frac{\text{cov}(\Delta x_1, \Delta x_2)}{\text{var}(\Delta x_1)}\left[\Delta x_1 - E(\Delta x_1)\right] \\
\text{var}(\Delta x_2 \mid \Delta x_1) &= \text{var}(\Delta x_2)\left[1 - \frac{(\text{cov}(\Delta x_1, \Delta x_2))^2}{\text{var}(\Delta x_1)\text{var}(\Delta x_2)}\right].
\end{aligned}$$

Because the contestants are *ex ante* identical and the first-period equilibrium is symmetric we know that $E(\Delta x_1) = 0$.

Hence the first-order conditions are given by

$$\begin{aligned}
ke_2^A &= \frac{\phi(y_2)}{\sqrt{\text{var}\,(\Delta x_2 \mid \Delta x_1)}} \\
&\quad \times \left\{\mu + \frac{\sigma^2 e_1}{\text{var}(\Delta x_1)}\Delta x_1 - \frac{\sigma^2\left[e_2^A - \frac{\text{cov}(\Delta x_1, \Delta x_2)}{\text{var}(\Delta x_1)}e_1\right]}{\text{var}\,(\Delta x_2 \mid \Delta x_1)}T\right\} \\
ke_2^B &= \frac{\phi(y_2)}{\sqrt{\text{var}\,(\Delta x_2 \mid \Delta x_1)}} \\
&\quad \times \left\{\mu - \frac{\sigma^2 e_1}{\text{var}(\Delta x_1)}\Delta x_1 + \frac{\sigma^2\left[e_2^B - \frac{\text{cov}(\Delta x_1, \Delta x_2)}{\text{var}(\Delta x_1)}e_1\right]}{\text{var}\,(\Delta x_2 \mid \Delta x_1)}T\right\},
\end{aligned}$$

where $T = \Delta x_1 + E\,(\Delta x_2 \mid \Delta x_1)$.

References

Aoyagi, M., 2007, "Information Feedback in a Dynamic Tournament," Discussion Paper No. 580, The Institute of Social and Economic Research, Osaka University.

Ashford, S.J., 1986, "Feedback-Seeking in Individual Adaptation: A Resource Perspective," *Academy of Management Journal*, 29, 465–487.

Baye, M.R., D. Kovenock, and C.G. De Vries, 1993, "Rigging the Lobbying Process: An Application of All-Pay Auctions," *American Economic Review*, 83, 289–294.

Beer, M., 1987, "Performance Appraisals," in J. Lorsch, ed., *Handbook of Organizational Behavior*, Englewood Cliffs, NJ: Prentice Hall, Inc., 286–301.

DeVries, D.L., A.M. Morrison, S.L. Shullman, and M.L. Gerlach, 1986, *Performance Appraisal on the Line*, Greensboro, NC: Center for Creative Leadership.

Ederer, F. and E. Fehr, 2007, "Deception and Incentives: How Dishonesty Undermines Effort Provision," IEW Working Paper 341, Institute for Empirical Research in Economics, University of Zürich.

Efron, B., 1965, "Increasing Properties of Polya Frequency Functions," *Annals of Mathematical Statistics*, 36, 272–279.

Fang, H. and G. Moscarini, 2005, "Morale Hazard," *Journal of Monetary Economics*, 52(4), 749–778.

Fuchs, W., 2007, "Contracting with Repeated Moral Hazard and Private Evaluations," *American Economic Review*, 97(4), 1432–1448.

Fudenberg, D. and J. Tirole, 1986, "A "Signal-Jamming" Theory of Predation," *RAND Journal of Economics*, 17(3), 366–376.

Gershkov, A. and M. Perry, 2007, "Tournaments with Midterm Reviews," *Games and Economic Behavior*, 66(1), 162–190.

Gibbs, M., 1991, "An Economic Approach to Process in Pay and Performance Appraisals," manuscript, University of Chicago, Graduate School of Business.

Goltsman, M. and A. Mukherjee, 2008, "Information Disclosure in Multistage Tournaments," Mimeo, Michigan State University, Ann Arbor.

Green, E. and N. Stokey, 1983, "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, 91, 349–364.

Grossman, S.J., 1981, "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics*, 24(3), 461–483.

Holmstrom, B., 1982, "Managerial Incentive Problems—A Dynamic Perspective," in *Essays in Economics and Management in Honor of Lars Wahlbeck*, Helsinki: Swedish School of Economics (see also *Review of Economic Studies*, January 1999).

Jewitt, I., 1985, "Risk and Risk Aversion in the Two Risky Asset Portfolio Problem," Mimeo, Nuffield College, University of Oxford.

Jung, W. and Y.K. Kwon, 1988, "Disclosure When the Market Is Unsure of Information Endowment of Managers," *Journal of Accounting Research*, 26(1), 146–153.

Lazear, E. and S. Rosen, 1981, "Rank Order Tournaments as Optimal Labor Contracts," *Journal of Political Economy*, 89, 841–864.

Leland, H.E., 1968, "Saving and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics*, 82(3), 465–473.

Levin, J. and S. Tadelis, 2005, "Profit Sharing and the Role of Professional Partnerships," *Quarterly Journal of Economics*, 120(1), 131–172.

Liden, R.C. and T.R. Mitchell, 1985, "Reactions to Feedback: The Role of Attributions," *Academy of Management Journal*, 28, 291–308.

Lizzeri, A., M.A. Meyer, and N. Persico, 1999, "Interim Evaluations in Dynamic Tournaments: The Effects of Midterm Exams," unpublished manuscript.

——, ——, and ——, 2002, "The Incentive Effects of Interim Performance Evaluations," CARESS Working Paper #02-09, University of Pennsylvania, College Park.

Locke, E. and G. Latham, 1990, *A Theory of Goal Setting and Task Feedback*, Englewood Cliffs, NJ: Prentice Hall.

Manso, G., 2007, "Motivating Innovation," Mimeo, MIT Sloan, Cambridge.

Meyer, M.A., 1991, "Learning from Coarse Information: Biased Contests and Career Profiles," *Review of Economic Studies*, 58, 15–41.

——, 1992, "Biased Contests and Moral Hazard: Implications for Career Profiles," *Annales d'Économie et de Statistique*, 25/26, 165–187.

Meyer, H.H., E. Kay, and J.R. French, 1965, "Split Roles in Performance Appraisal," *Harvard Business Review*, 43, 123–129.

Milgrom, P.R., 1981, "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12(2), 380–391.

—— and R. Weber, 1982, "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1059–1122.

Moore, D.A. and W.M.P. Klein, 2008, "Use of Absolute and Comparative Performance Feedback in Absolute and Comparative Judgments and Decisions," *Organizational Behavior and Human Decision Processes*, 107(1), 60–74.

Morgan, J. and V. Krishna, 1997, "An Analysis of the War of Attrition and the All-Pay Auction," *Journal of Economic Theory*, 72, 343–362.

Murphy, K.R. and J.N. Cleveland, 1995, *Performance Appraisal: An Organizational Perspective*, Boston: Allyn and Bacon.

Nalebuff, B.J. and J.E. Stiglitz, 1983, "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, 14(1), 21–43.

O'Keefe, M., W.K. Viscusi, and R.J. Zeckhauser, 1984, "Economic Contests: Comparative Reward Schemes," *Journal of Labor Economics*, 2(1), 27–56.

Podsakoff, P.M. and J. Farh, 1989, "Effects of Feedback Sign and Credibility on Goal Setting and Task Performance," *Organizational Behavior and Human Decision Processes*, 44(1), 45–67.

Rosen, S., 1986, "Prizes and Incentives in Elimination Tournaments," *American Economic Review*, 76(4), 701–715.

Schotter, A. and K. Weigelt, 1992, "Asymmetric Tournaments, Equal Opportunity Laws, and Affirmative Action: Some Experimental Results," *Quarterly Journal of Economics*, 107(2), 511–539.

Shin, H.S., 1994, "News Management and the Value of Firms," *RAND Journal of Economics*, 25(1), 58–71.

Siegel, R., 2009, "All-Pay Contests," *Econometrica*, 77(1), 71–92.

Stone, D.L. and E.F. Stone, 1985, "The Effects of Feedback Consistency and Feedback Favorability on Self-Perceived Task Competence and Perceived Feedback Accuracy," *Organizational Behavior and Human Decision Processes*, 36(2), 167–185.

Yildirim, H., 2005, "Contests with Multiple Rounds," *Games and Economic Behavior*, 51, 213–227.